MA 243 HOMEWORK 5

DUE: THURSDAY, 13 NOVEMBER, 2008, BY 12PM

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

(1) Use the spherical cosine law to compute the distance between the points $(1, 0, 0)$ and $(0, 1/\sqrt{2}, 1/\sqrt{2})$ on S^2.
(2) Describe all motions of $\mathbb{E}^2$ you can obtain by repeatedly reflecting in the y axis and reflecting in the line $x = 1$.
(3) Recall that $\mathbb{H}^1 = \{(t, x) \in \mathbb{R}^2 : -t^2 + x^2 = -1\}$. Show that the map $\phi : \mathbb{R} \to \mathbb{H}^1$ given by $\phi(s) = (\cosh(s), \sinh(s))$ is a bijection.
(4) Prove the hyperbolic double-angle formula:

$$\cosh(\beta + \gamma) = \cosh(\beta)\cosh(\gamma) + \sinh(\beta)\sinh(\gamma).$$

(5) Write a list summarizing in just one or two sentences the content of each lecture so far.

B: Exercises

(1) Use the main formula of spherical trig to calculate the distance from London to Christchurch, NZ on the surface of the earth, using that London is approximately 51° North, and Christchurch is approximately 43° South, 172° East. Recall that latitude is measured from the equator 0° north to the North Pole = 90° N, and longitude is measured from the Greenwich observatory, which is in London. The circumference of the earth is 40,000 km by the definition of kilometer.
(2) Prove that $P, Q, R \in S^2$ are collinear if and only if either $d(P, Q) + d(Q, R) = d(P, R)$ after relabelling, or $d(P, Q) + d(Q, R) + d(P, R) = 2\pi$.
(3) Show that if $T(x) = Ax$ is a linear map from $\mathbb{R}^2$ to itself (so A is a 2×2 matrix) with the property that T maps $\mathbb{H}^1$ to itself and

preserves distance, then A has one of the following two forms:

$$A = \begin{pmatrix} \cosh(s) & \sinh(s) \\ \sinh(s) & \cosh(s) \end{pmatrix}, \quad A = \begin{pmatrix} \cosh(s) & -\sinh(s) \\ \sinh(s) & -\cosh(s) \end{pmatrix}$$

(4) Show that if L is a hyperbolic line then there is a distance preserving bijection from L to $\mathbb{H}^1$.

C: Extensions

(1) (a) Consider the motions of $\mathbb{E}^2$ given by the reflection T in the x-axis and the reflection S in the y-axis. How many different motions of $\mathbb{E}^2$ can you obtain by repeated composition of T and S? (for example, $T \circ S$, $T \circ S \circ T \circ S \circ S$). line $y = -\sqrt{3}x$?

(b) Consider the motions of $\mathbb{E}^2$ given by the reflection T in the x-axis and the reflection S in the line $y = \tan(2\pi/n)$ for a fixed $n \geq 3$. How many different motions of $\mathbb{E}^2$ can you obtain by repeated composition of T and S? How does your answer change if S is a general line $y = cx$ for a fixed c not equal to $\tan(\pi/n)$ for some n? What if the two lines of reflection do not intersect?

(2) Do any (all!) of the exercises on the sphere in Chapter 3 of the book.