MA 243 HOMEWORK 4

DUE: THURSDAY, OCTOBER 30, 2008

Hand in the problems in Section B *only* to the boxes outside the undergraduate office. You are encouraged to work together on the problems, but your written work should be your own.

A : Warm-up problems

1. Decompose the motion
$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
of $\mathbb{E}^2$ into a product of reflections.
2. Show that every rotation in $\mathbb{E}^2$ can be written as the composition of two reflections. (Hint: Last week's homework).

B: Exercises

1. Prove that if triangles ABC and $A'B'C'$ have $d(A, B) = d(A', B')$, and the angles at A and B equal the angles at A' and B' respectively: $\angle BAC = \angle B'A'C', \angle ABC = \angle A'B'C'$, then ABC is congruent to $A'B'C'$. Use the language and definitions of this module.
2. Let T be the motion of $\mathbb{E}^3$ given in coordinates by $T(x_1, x_2, x_3) = (\mathbf{x}) = (-x_3, -x_2, x_1)$. Write T as the composition of rotations and reflections and a translation as described in class. Then write T as the composition of at most four reflections as described in class.
3. Recall that the perpendical bisector of $P, Q \in \mathbb{E}^n$ is the set $\{R \in \mathbb{E}^n : d(P, R) = d(Q, R)\}$. Show that this is an affine hyperplane in $\mathbb{E}^n$.
4. Write an equation for the perpendicular bisector Π of the line between $(2, 0, 0)$ and $(2, 1, 3)$. Write down in coordinates the motion of reflecting in the plane Π.

C: Extensions

1. (a) Show that any motion of $\mathbb{E}^n$ is invertible.
   (b) If T is a rotation in $\mathbb{E}^2$, what is the inverse?

(c) If S is a translation in $\mathbb{E}^2$, what is the inverse?
(d) If U is a reflection in $\mathbb{E}^2$, what is the inverse?
(e) With T, S, U as above, what is $S^{-1} \circ T \circ S$? What about $S^{-1} \circ U \circ S$?
(f) (Extension part, for those who know more group theory) This is showing that the *group of orthogonal matrices* (motions fixing a particular point P) is a *normal subgroup* of the *group of all Euclidean motions*, and in fact the group of all motions is a semi-direct product of the orthogonal motions and the translations. Understand this sentence! (More realistically, come back to this question once you learn what a semi-direct product is).