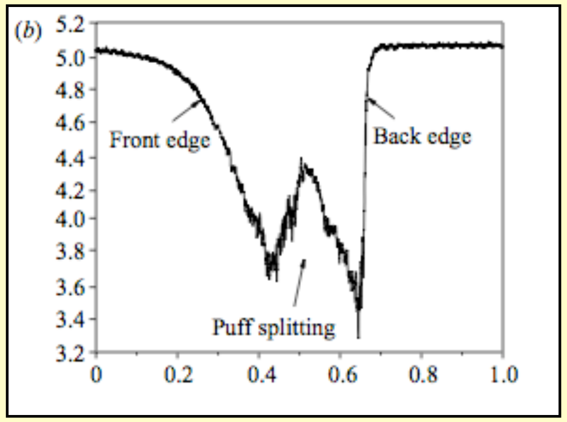
Simplifying the Complexity of Pipe Flow

Transitional Pipe Flow

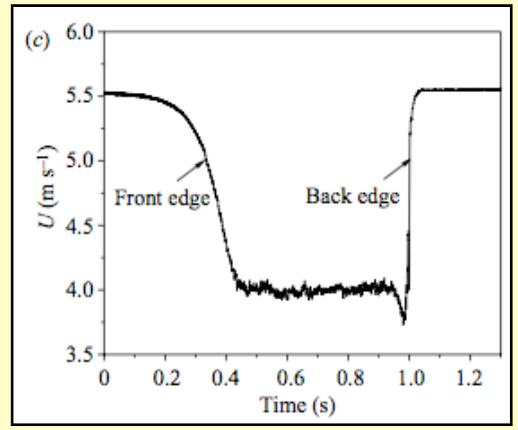
(From the work of many)

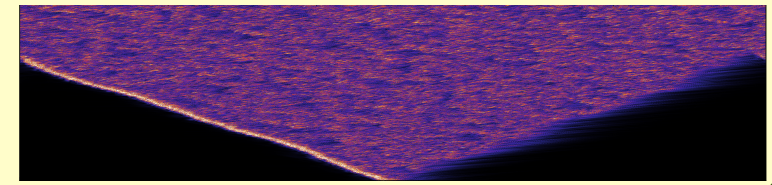
Nishi, *et al*. (JFM, 2008)

u



time

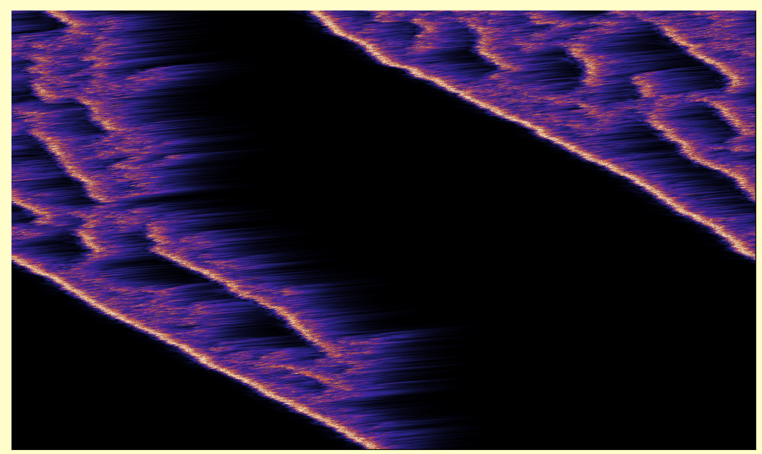




Re

Moxey, Barkley (PNAS 2010)

time



x - Ut

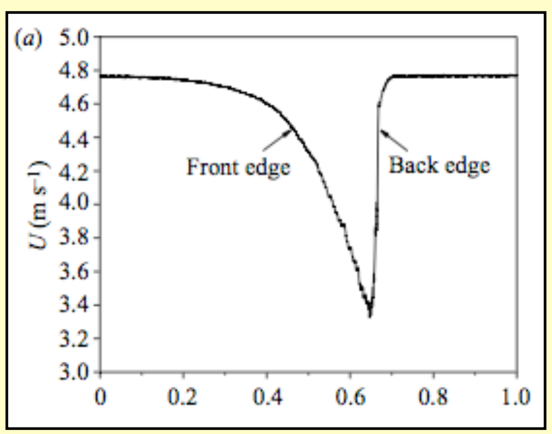
slugs

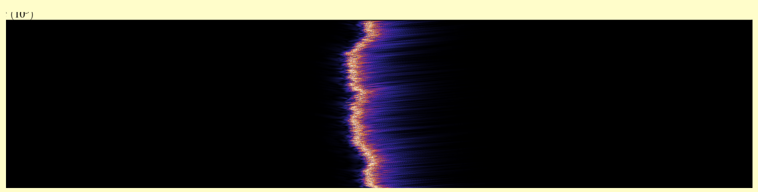
(~uniform)

~2600

puff splitting

(intermittency)





~2035

“equilibrium” puffs

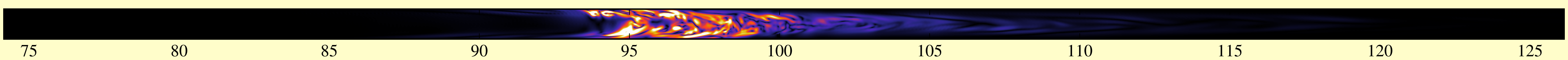
(long-time transient)

Physical Ideas

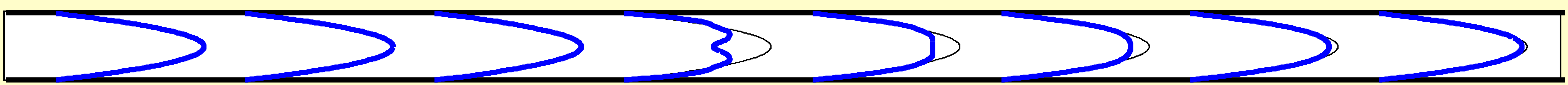
(Laufer (1962) → *et al.* → Hof *et al.* (2010))

- Fluid continually in transitional state, moving into and out of turbulence.
- Turbulence invades <u>upstream</u> laminar flow.
- Transition to turbulence fast, recovery of shear profile is slow.

motion in lab frame →



← motion relative to ~ mean or bulk velocity



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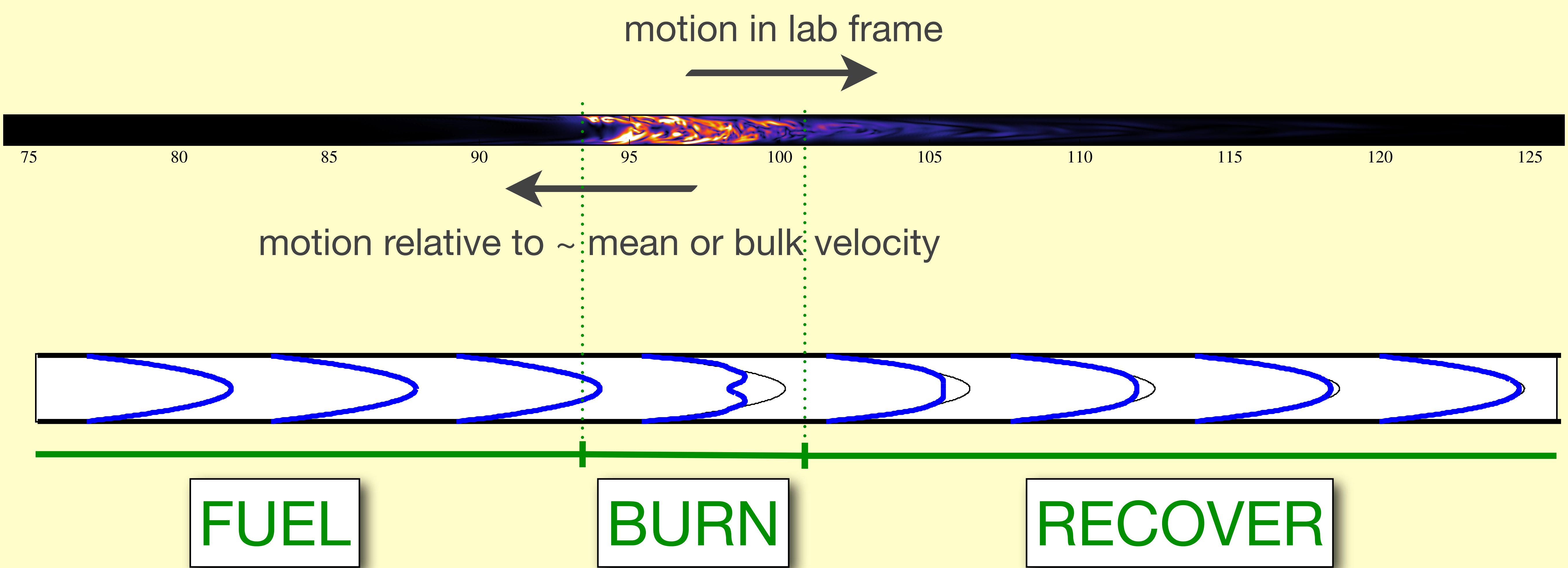

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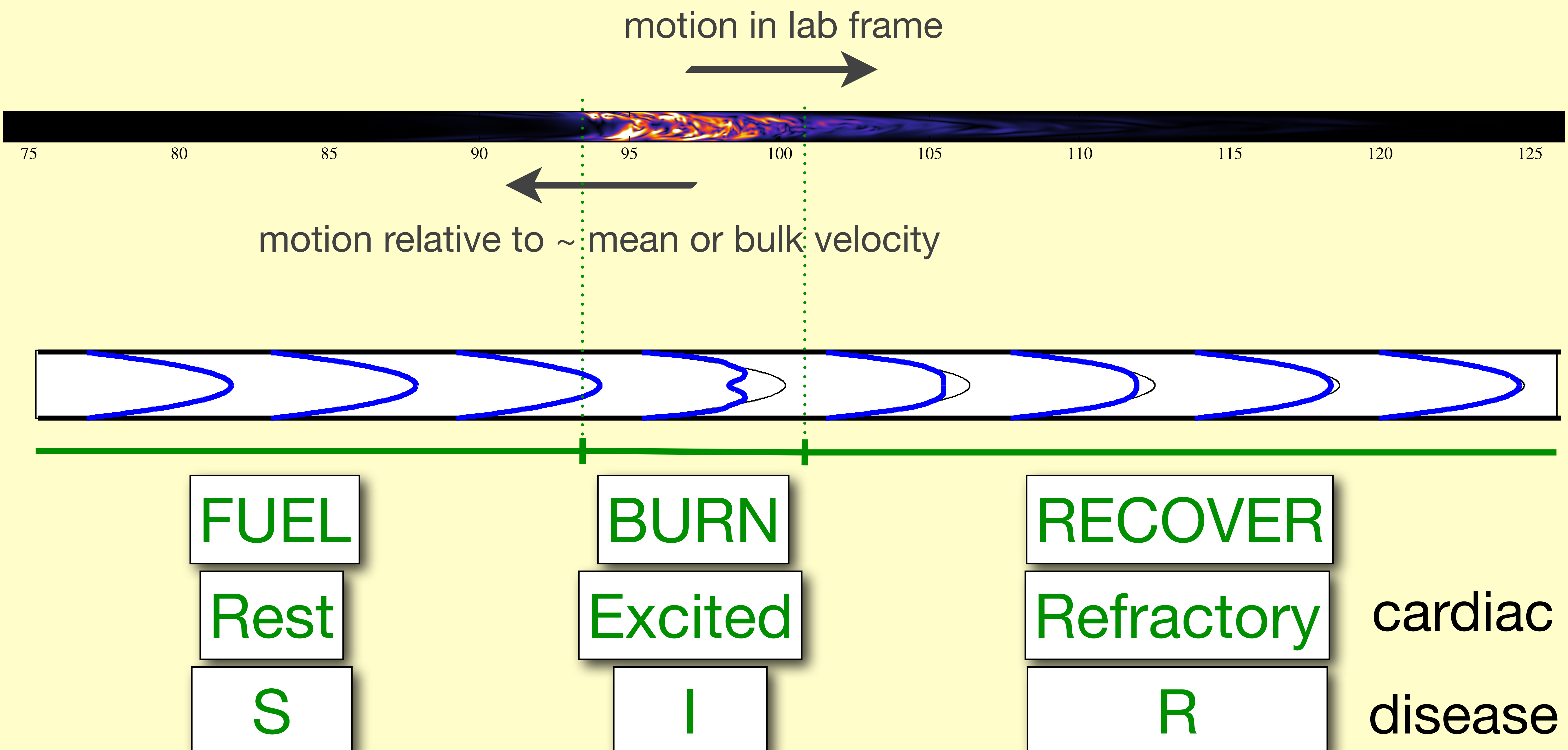


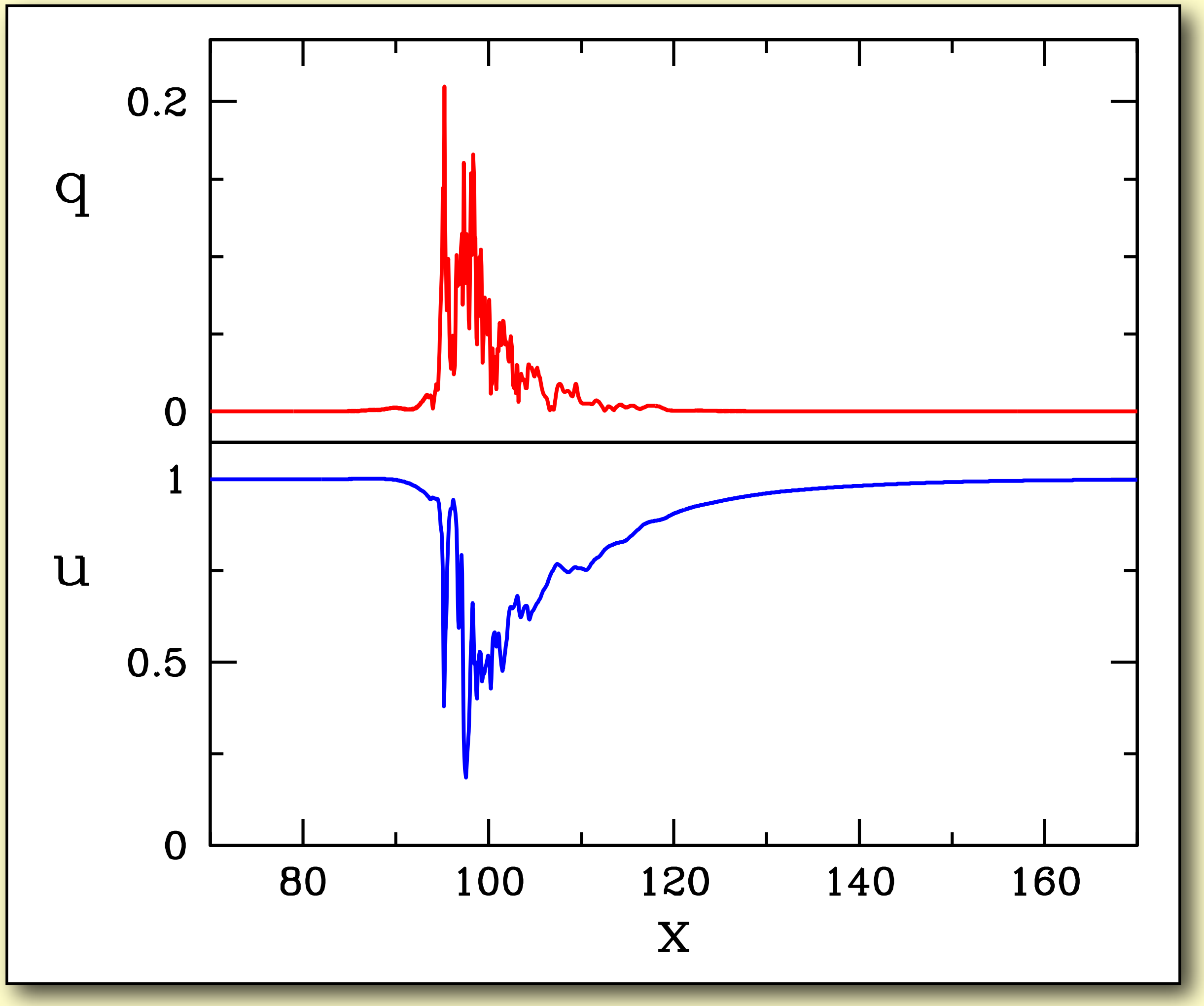
Minimum Variables

- Turbulence intensity: q
- Axial velocity on centerline (relative to mean): u

puff from simulations

Magnitude of transverse velocity on centerline

(axial velocity) - (mean velocity) on centerline



Turbulence Intensity

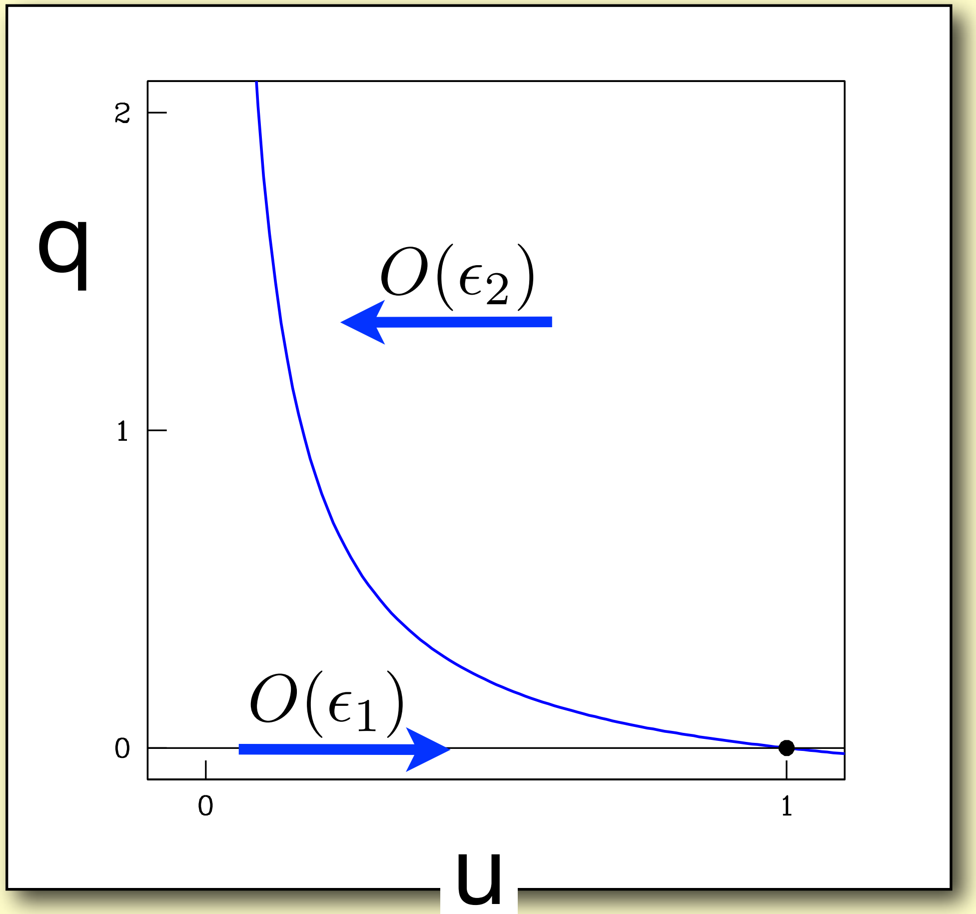
Proxy for shear, captures slow recovery

PDE Model

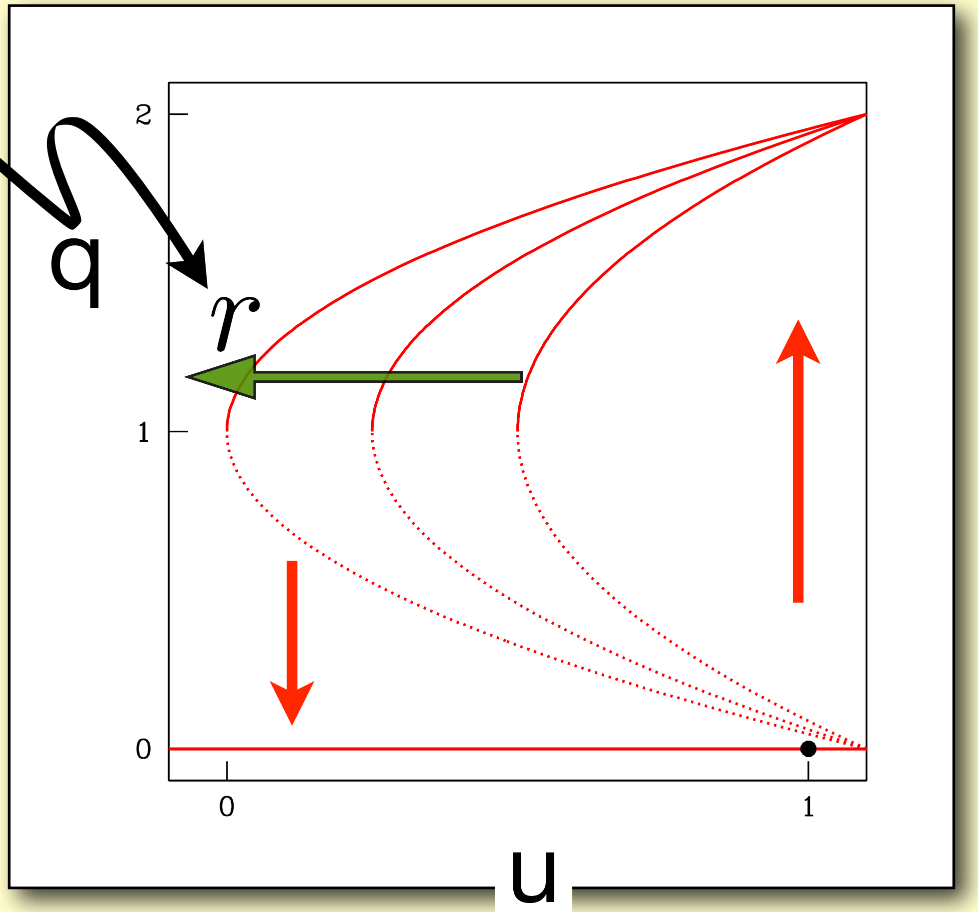
$$\partial_t u = \epsilon_1(1-u) - \epsilon_2 q u - \partial_x u$$
$$\partial_t q = q\left(u + r - 1 - (r+\delta)(q-1)^2\right) + \partial_{xx} q$$

"Reynolds number"

u dynamics

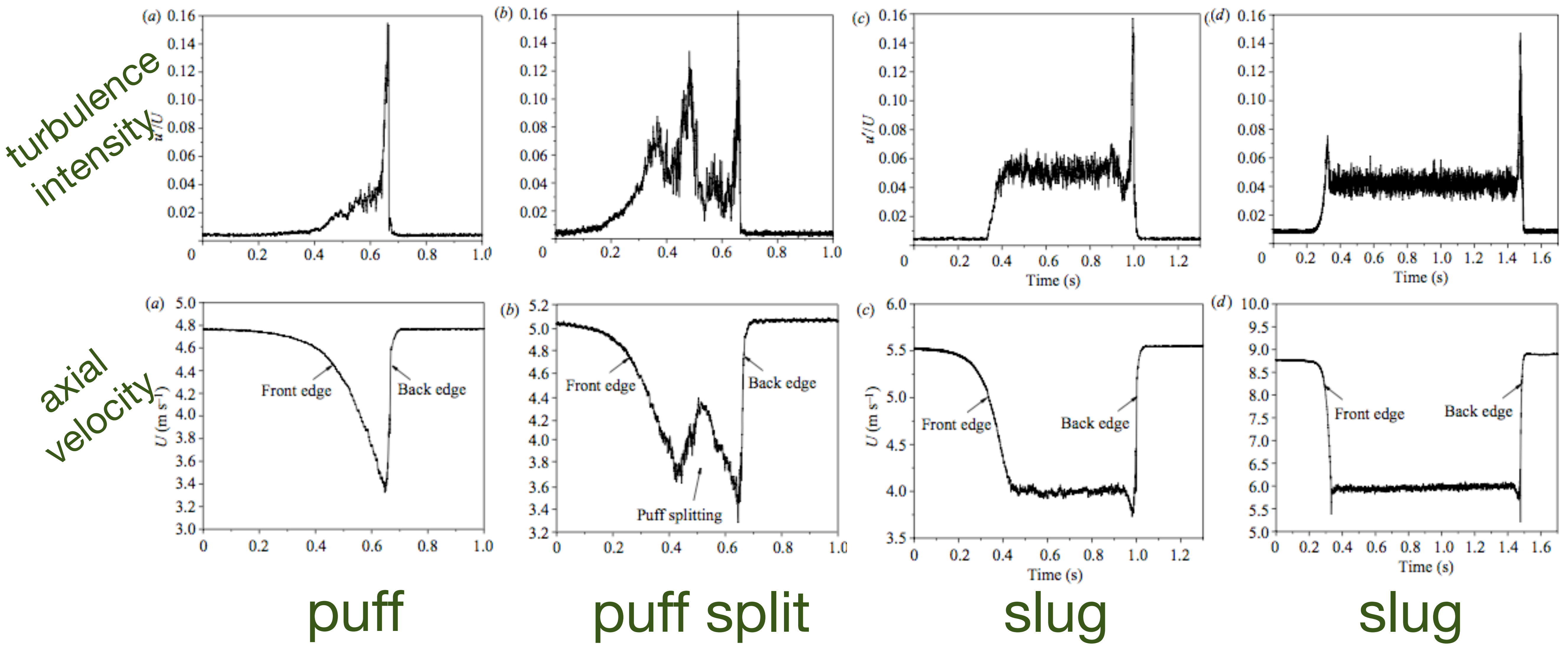


q dynamics



Plus linear advection $\partial_x u$ and diffusive coupling $\partial_{xx} q$
(SO(2) Symmetry)

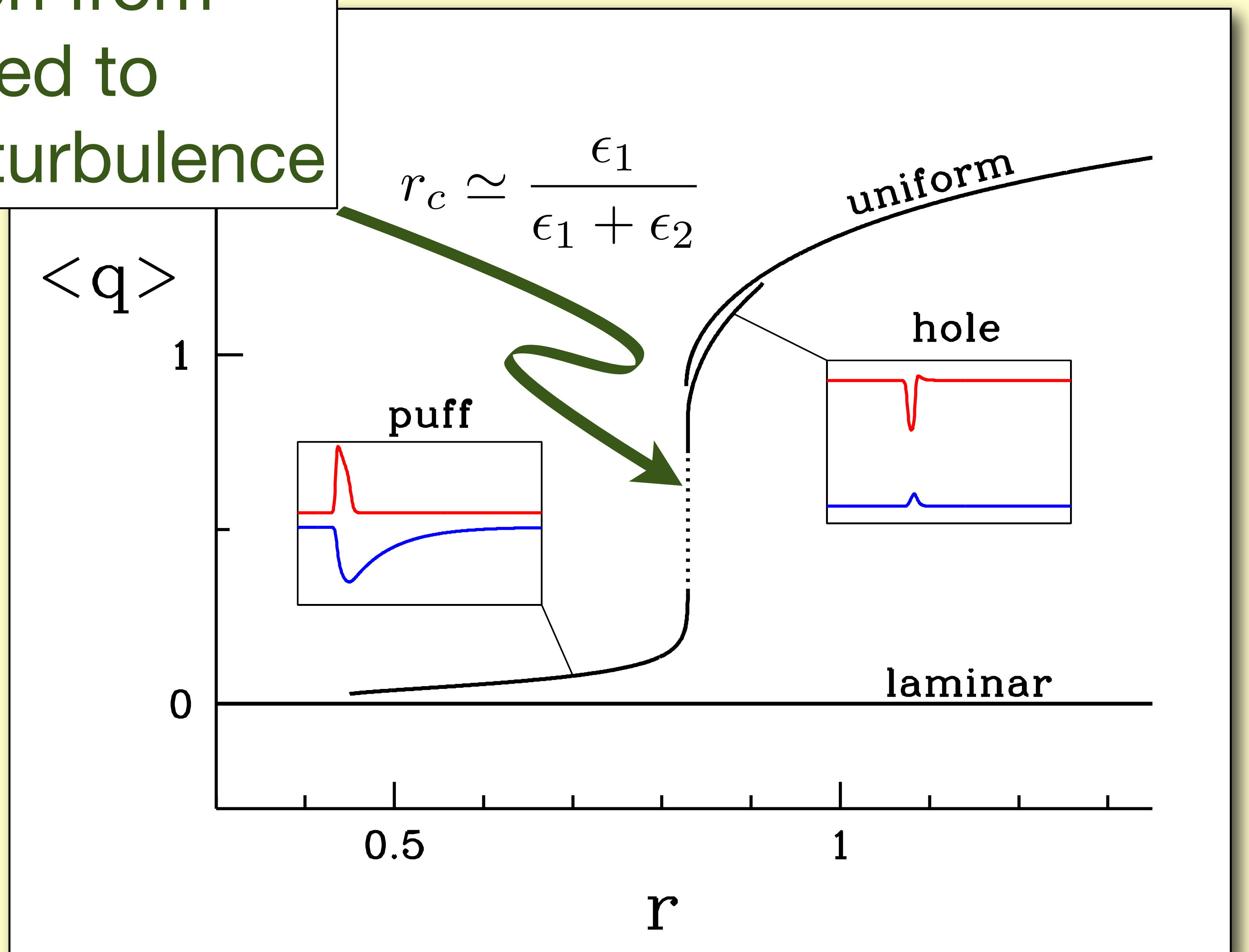
M. Nishi, B. Ünsal, F. Durst and G. Biswas



Analysis of PDE Model

Standard numerical and analytical techniques give complete picture

Transition from localized to expanding turbulence



Check List

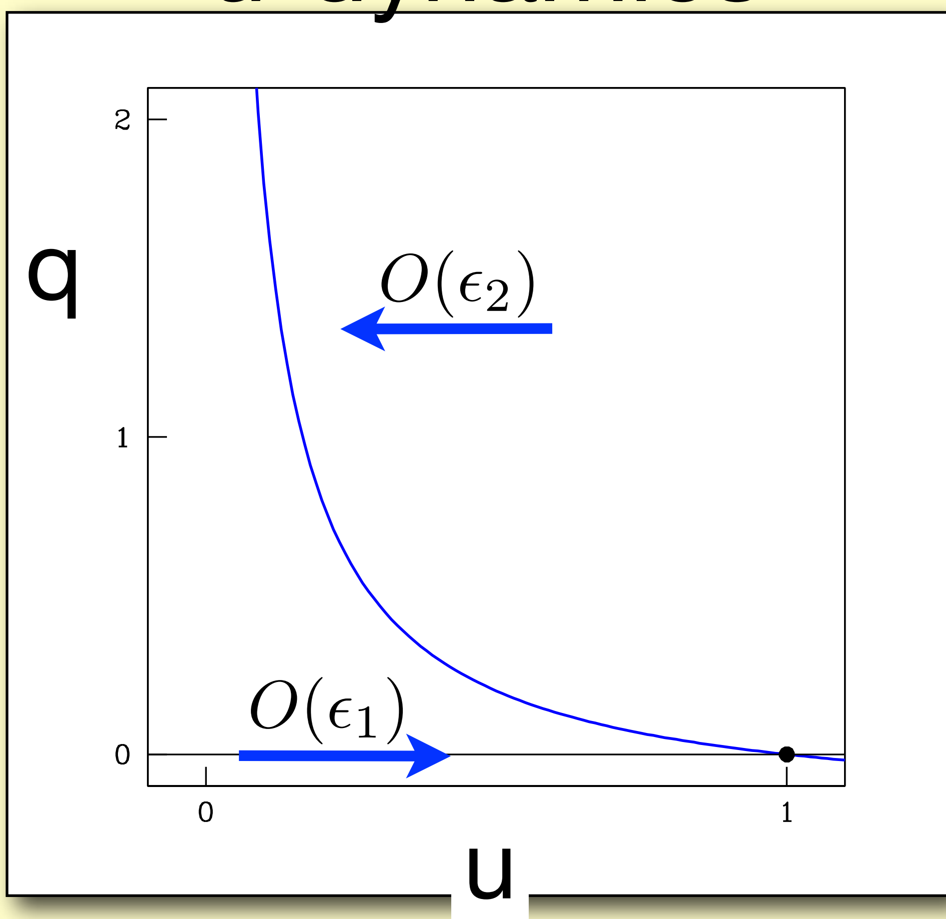
- [x] laminar state stable at all r
- [x] localized puffs
- [x] expanding turbulence
- [x] localized edge states
- [ ] finite-lifetime puffs
- [ ] puff splitting

Discrete Model

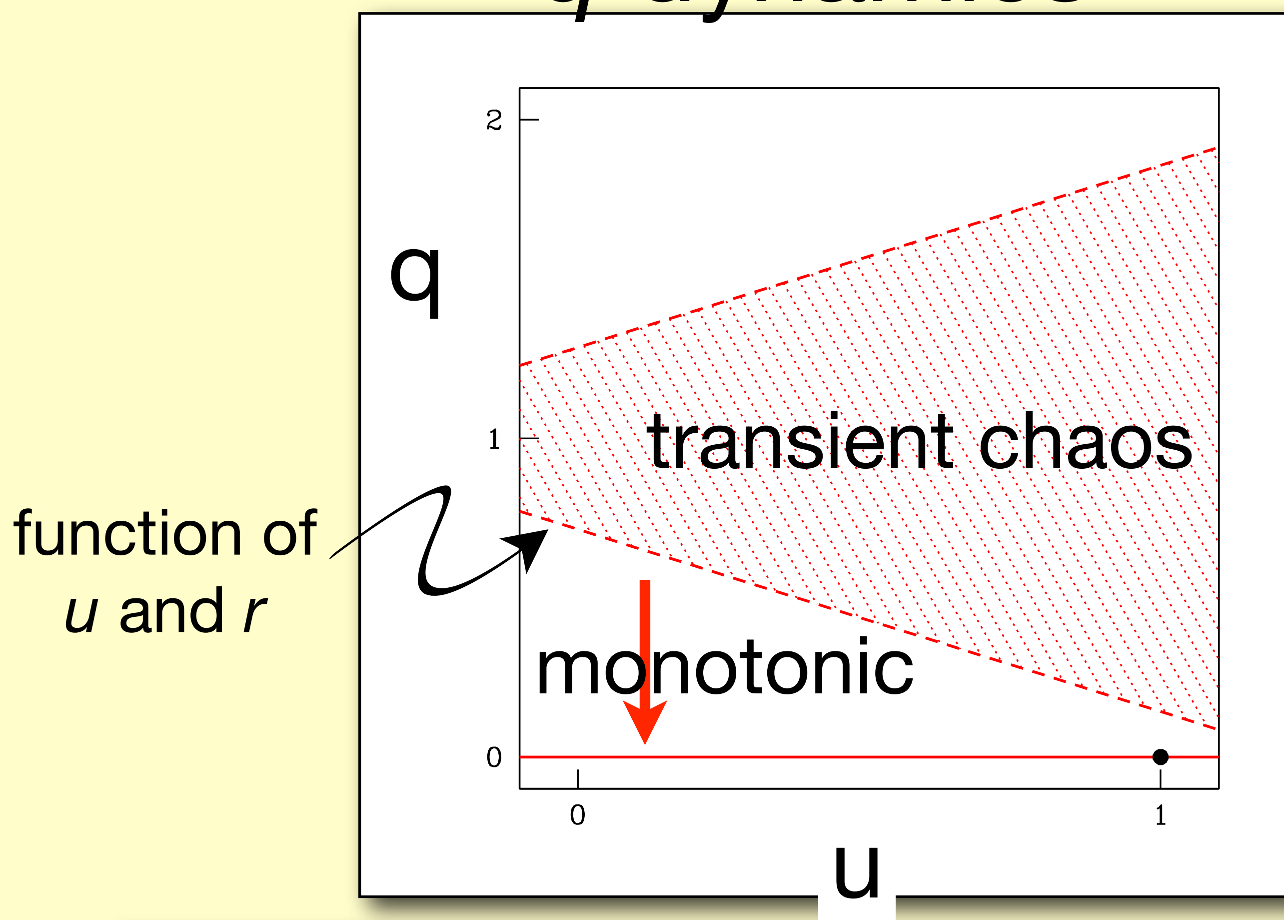
$$u_i^{n+1} = u_i^n + \epsilon_1(1 - u_i^n) - \epsilon_2 u_i^n q_i^n - c(u_i^n - u_{i-1}^n)$$

$$q_i^{n+1} = g^k(u_i^{n+1}, q_i^n + d(q_{i-1}^n - 2q_i^n + q_{i+1}^n))$$

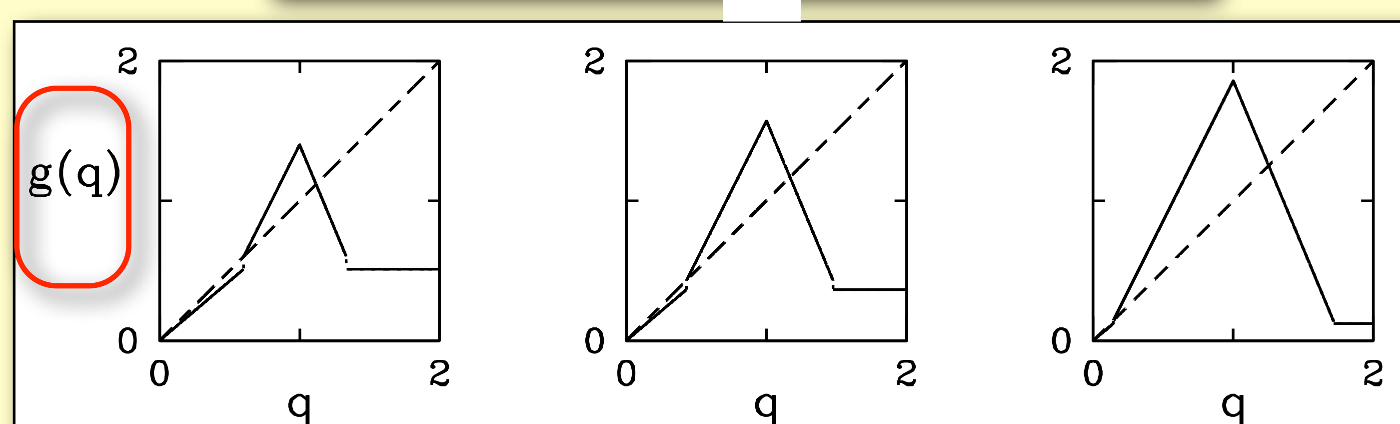
u dynamics



q dynamics



(c.f. Chate, Manneville *et al.*)

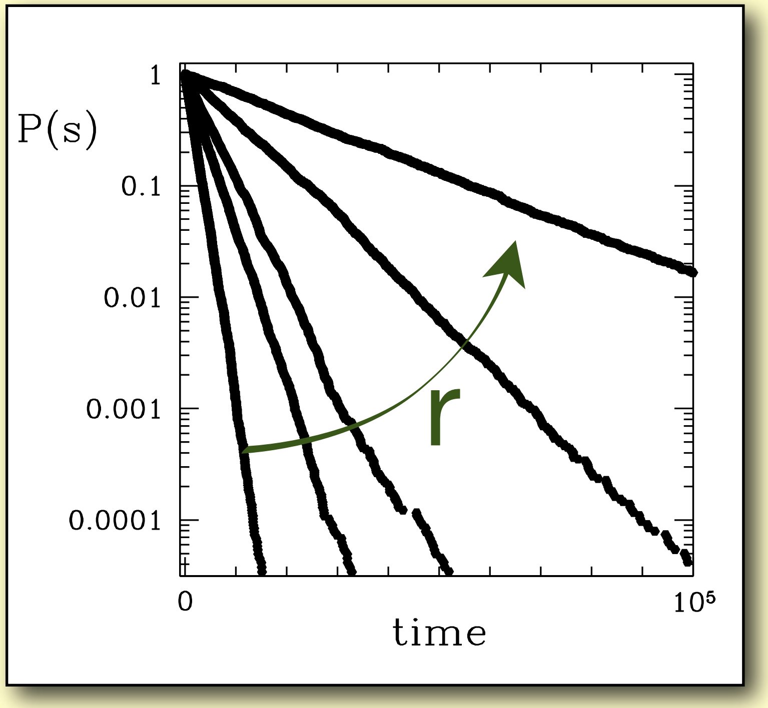


Quantitative Results

Decaying to Splitting Puffs

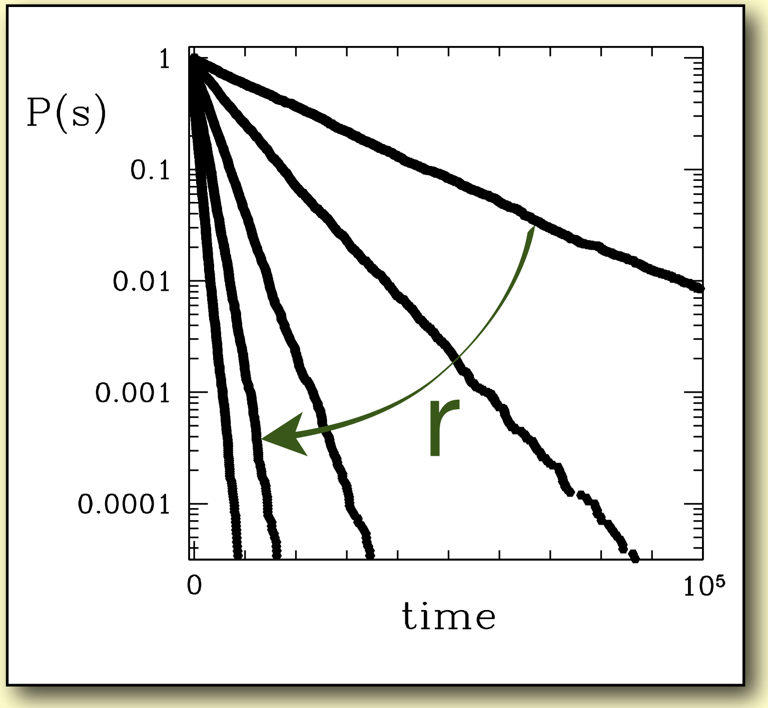
(c.f. K Avila, *et al*.)

Decay



Exponential probabilities of decay and splitting

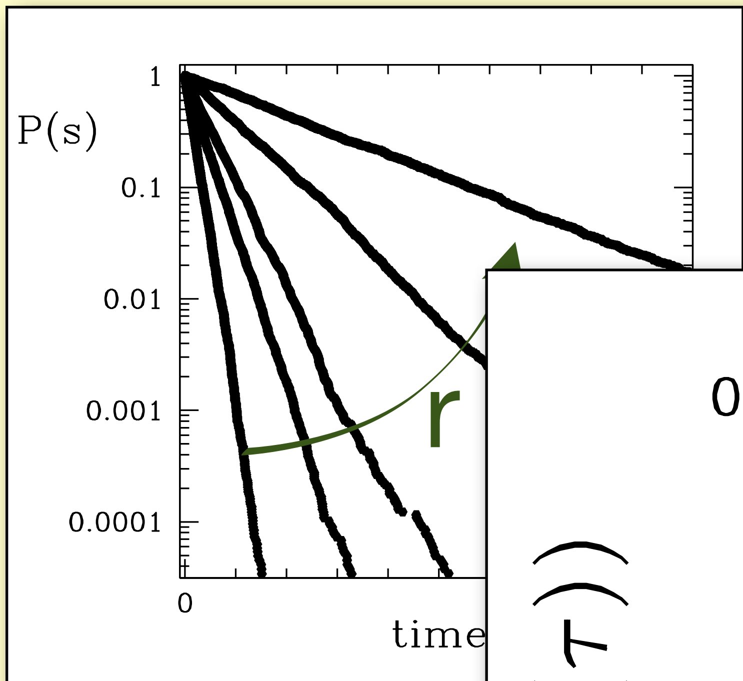
Split



Decaying to Splitting Puffs

(c.f. K Avila, *et al.*)

Decay



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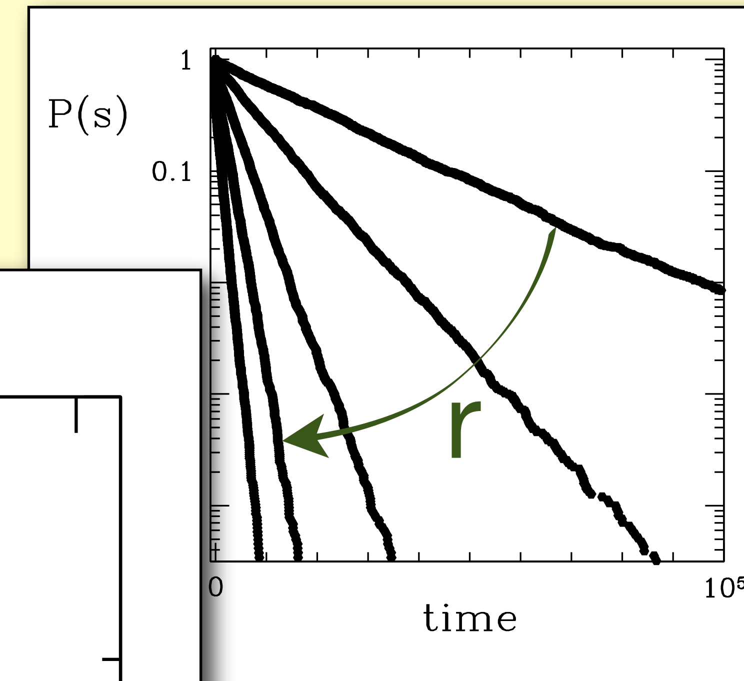
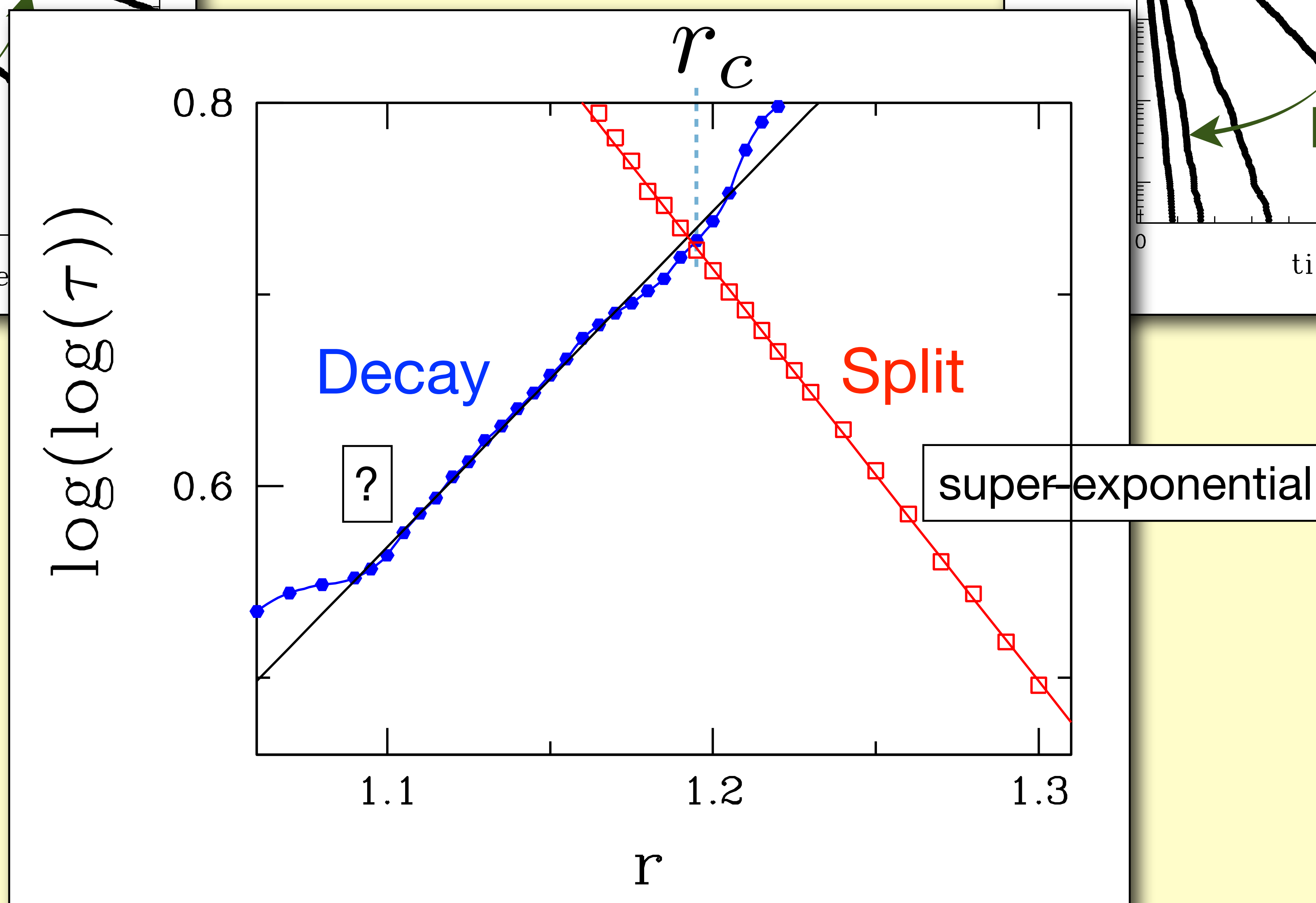
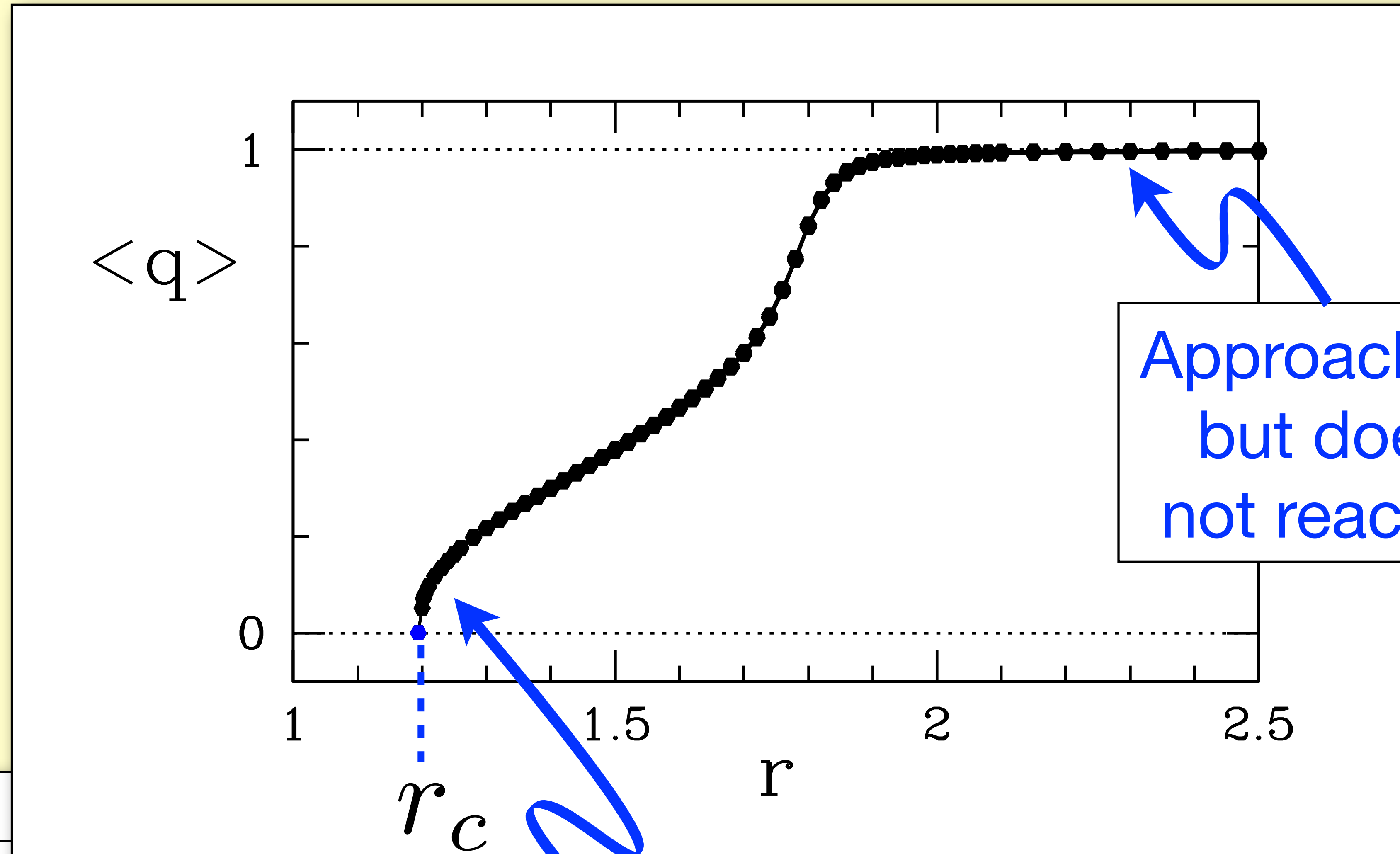




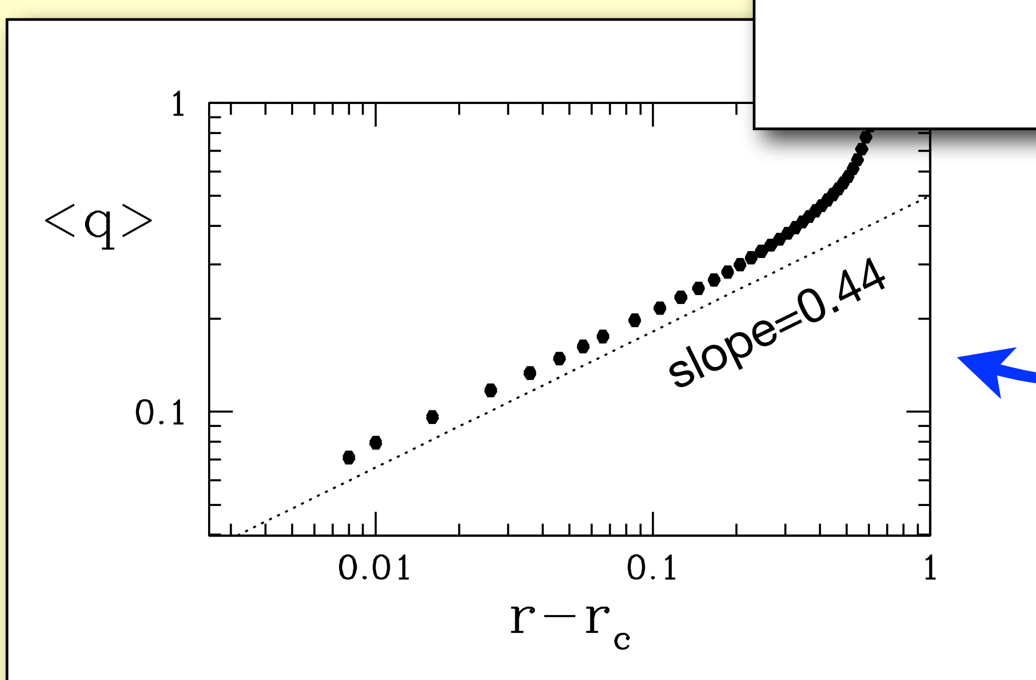
Equilibrium Turbulence Fraction

(c.f. M Avila, *et al.*)

mean over
X, T, E



Approaches,
but does
not reach 1



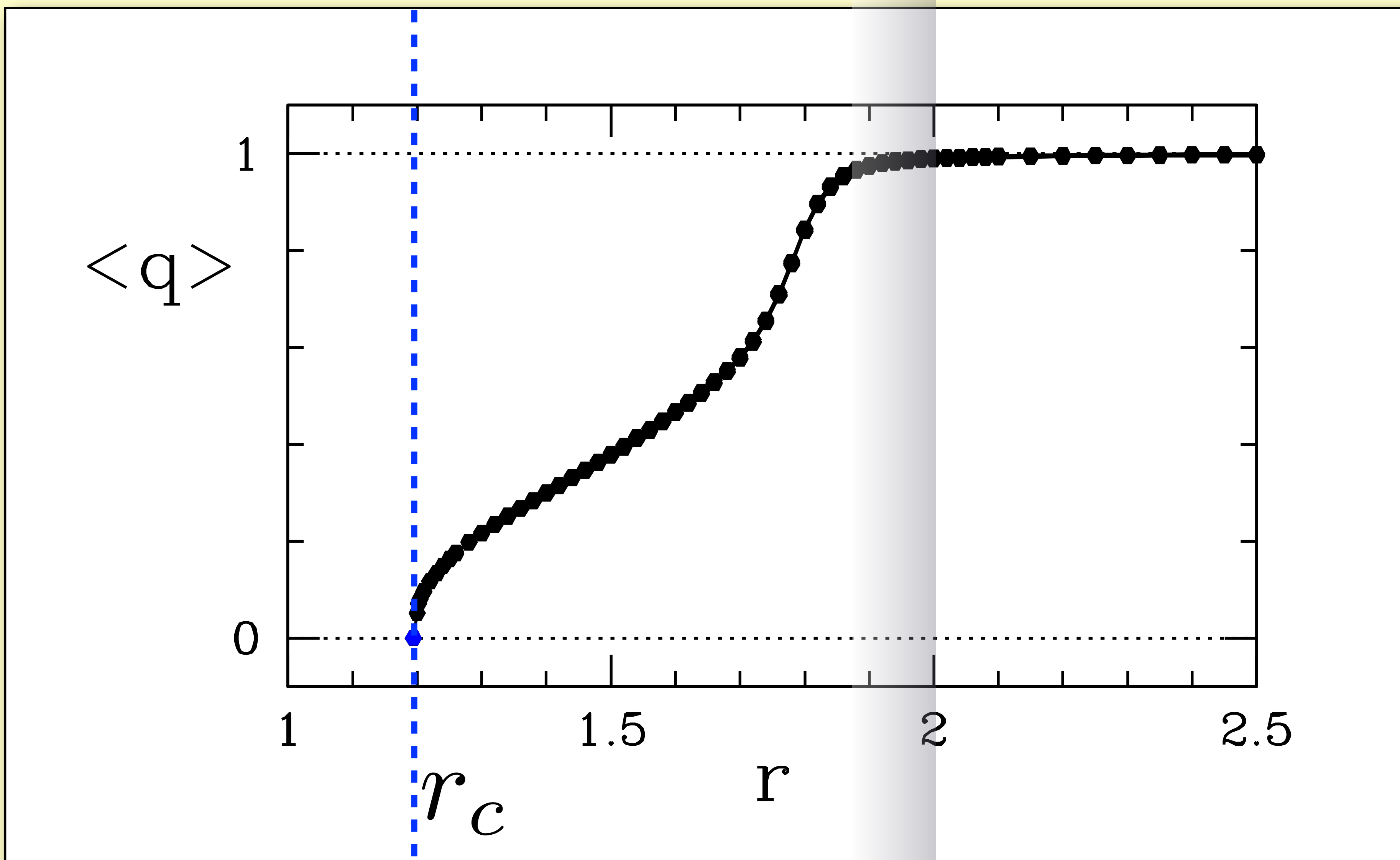
Continuous transition
to sustained turbulence
at r_c

Bifurcation Diagram

Transient

transient puffs | expanding turbulence

puff splitting | slugs



ST-Intermittency | ~ uniform turbulence

Asymptotic

laminar only | sustained turbulence

Check List

- [x] puffs lifetimes
  - [x] exponential distributions
  - [ ] super-exponential in r
- [x] puff splitting
  - [x] exponential distributions
  - [x] super-exponential in r
- [x] expanding turbulence & slugs
- [x] continuous transition to sustained turbulence at r_c
- [x] as r increases turbulence fraction → 1 laminar lengths → 0
- [x] localized edge states
- [x] unstable orbits extending to small r (below puffs)
- [x] puff interaction, i.e control

Discussion

- Simple considerations capture qualitatively almost all large-scale phenomena in transitional pipe flow.

- Helpful in not only in understanding pipe flow but also explaining why many flows (PCF, PPF, etc) show similar phenomena. Distinguish aspects specific to hydrodynamics (vortices and streaks) from generic aspects.

- Lack of super-exponential decay times is not necessarily a failure, it is interesting, but needs explanation.

FUTURE

- Comparison with experiment and DNS. Predictions.
- More quantitative modeling, but more importantly, should address Galilean invariance and boundary conditions.

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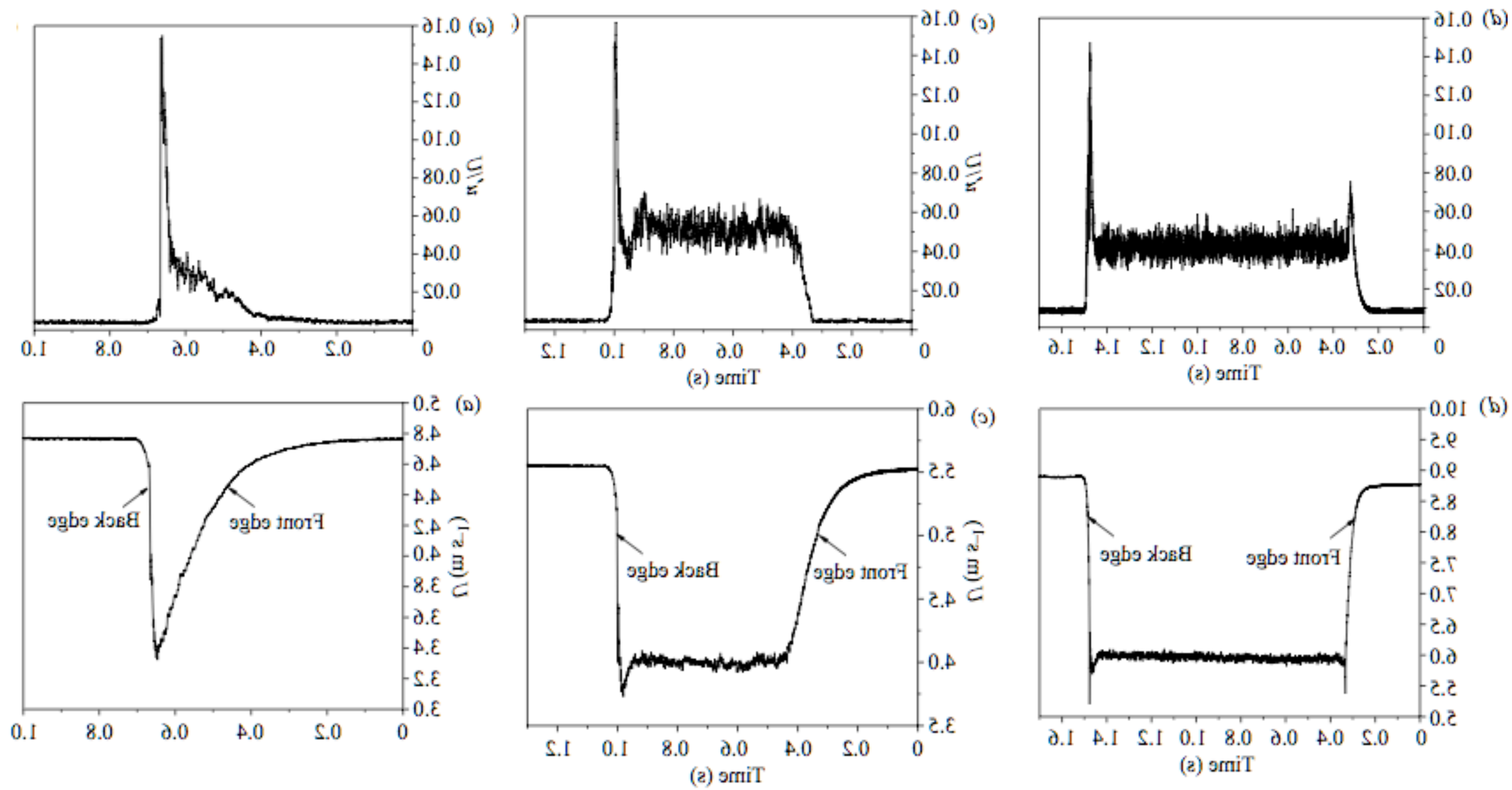
turbulence intensity

axial velocity



puff

puff split

slug

slug