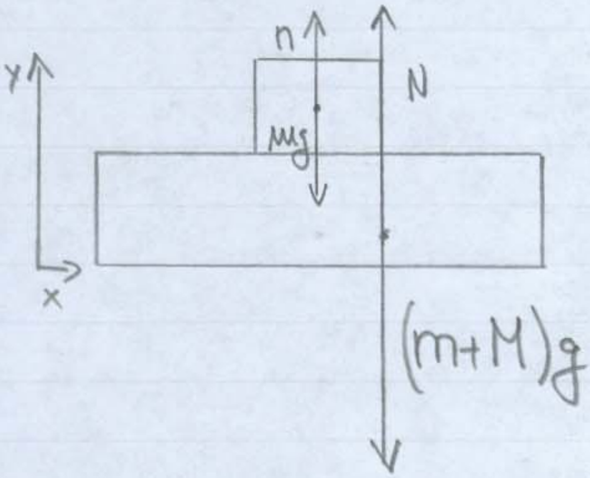
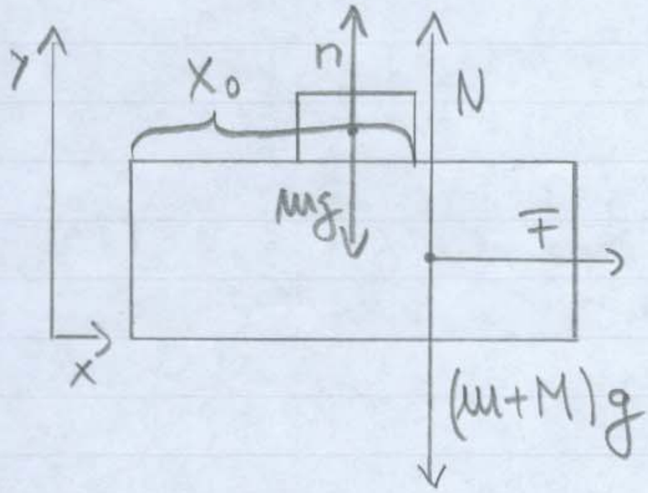
I 2 blocks on top of each other



$$n - mg = 0$$

$$N - (m+M)g = 0$$

II 2 blocks, lower on is pushed



$$n - mg = 0$$

$$N - (m+M)g = 0$$

$$F = MA$$

Since there is no friction between both blocks, the normal force will be $(m+M)g$ until the upper block falls off after

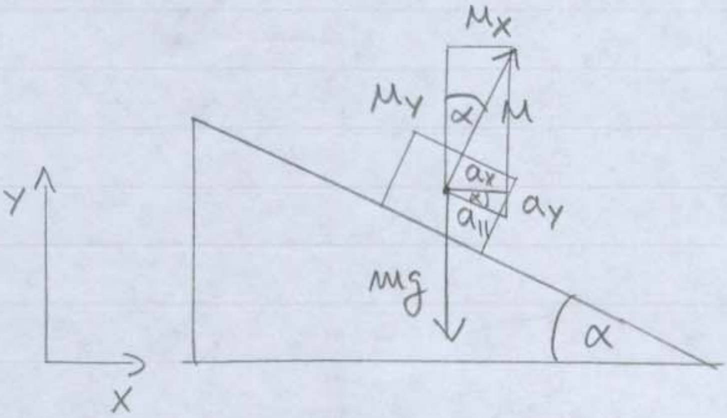
$$x_0 = \frac{1}{2} A t^2$$

$$\hookrightarrow \quad t = \sqrt{\frac{2x_0}{A}}$$

Then we have

$$F = MA, \quad N = Mg$$

III 2 blocks, lower is kept fixed



$$N_y - mg = -m a_y$$
$$N_x = m a_x$$

$$\sin\alpha = \frac{a_y}{a}, \quad \cos\alpha = \frac{a_x}{a} \qquad (a = a_{\parallel})$$

$$\frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha} = \frac{a_x}{a_y}$$

$$\tan\alpha = \frac{N_x}{N_y}$$

$$\frac{a_x}{a_y} = \frac{\frac{N_x}{m}}{g - \frac{N_y}{m}} = \frac{N_x}{mg - N_y}$$

$$\frac{a_y}{a_x} = \frac{mg - N_y}{N_x} = \frac{mg}{N_x} - \frac{N_y}{N_x}$$

$$\tan\alpha = \frac{mg}{N_x} - \frac{1}{\tan\alpha}$$

$$\hookrightarrow \frac{mg}{N_x} = \tan\alpha - \frac{1}{\tan\alpha}$$

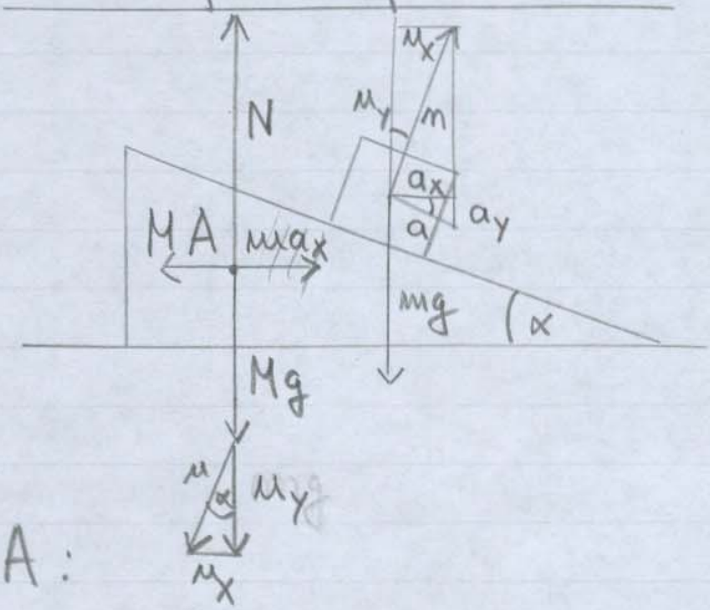
$$N_x = \frac{mg}{\tan\alpha - \frac{1}{\tan\alpha}} \overset{!}{=} m a_x$$

$$\hookrightarrow a_x = \frac{\cancel{m}g}{\cancel{m}\left(\tan\alpha - \frac{1}{\tan\alpha}\right)}$$

$$a = \frac{g/\cos\alpha}{\left(\tan\alpha - \frac{1}{\tan\alpha}\right)}$$

Test: $\alpha = 0 \Rightarrow a = \frac{g/1}{0 - \frac{1}{0}} = 0$ ✓

IV The full problem



$$\mu_y - mg = -ma_y$$
$$\mu_x = ma_x$$
$$N + (M+m)g = 0$$
$$MA = -ma_x$$

Part A:

From III, we have

$$a_x = a\cos\alpha = \frac{g}{\tan\alpha - \frac{1}{\tan\alpha}}$$

$$\hookrightarrow A = -\frac{m}{M}\left[\frac{g}{\tan\alpha - \frac{1}{\tan\alpha}}\right]$$

Part B:

$$a_x = \frac{g}{\tan\alpha - \frac{1}{\tan\alpha}}$$

Part C:

$$a_y = a\sin\alpha = \frac{g\tan\alpha}{\tan\alpha - \frac{1}{\tan\alpha}}$$

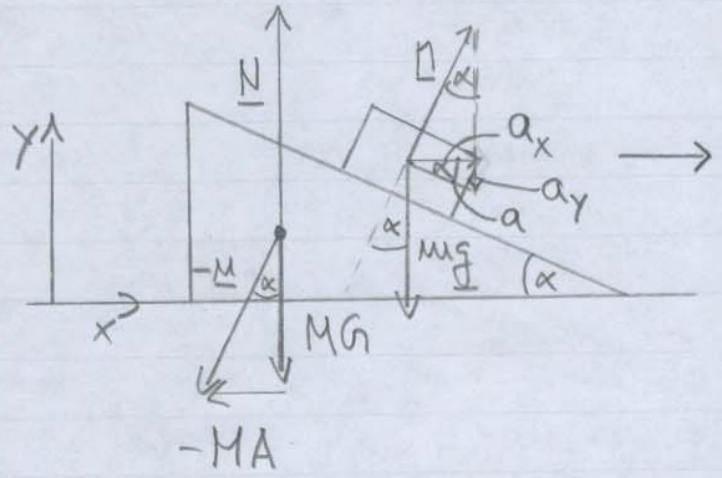
But this must be wrong, since a_x and a_y are independent of M.

Part D: add'tl force F on wedge in pos. x-direction

$$MA = -m a_x + F$$

$$\Rightarrow \quad m a_x = F - MA \stackrel{!}{=} 0$$

V The true solution



On the wedge:

$$MA = -\underbrace{\mu \sin\alpha}_{\mu_x} \qquad (1)$$

$$0 = N - MG - \underbrace{\mu \cos\alpha}_{\mu_y} \qquad (2)$$

On the block:

$$m a_x = \mu \sin\alpha \qquad (3)$$

$$+\, m a_y = \mu \cos\alpha - mg \qquad (4)$$

"a" is the acceleration of the block relative to the wedge. Thus $(a_x - A)$ is the true acceleration w.r.t. ground. Thus we write

$$+\, m(a_y - A) = \mu \cos\alpha - mg - mA$$

The block is moving rel. to the wedge. Thus

$$\mu - mg\cos\alpha - mA\sin\alpha = 0$$

is the condition that the block does not <u>sink</u> into the wedge.

Using (1), we can thus write

Part A:

$$-\frac{MA}{\sin\alpha} - mg\cos\alpha - mA\sin\alpha = 0$$

$$A\left(\frac{M}{\sin\alpha} + m\sin\alpha\right) = -mg\cos\alpha$$

$$\Rightarrow \quad A = \frac{-mg\cos\alpha}{\frac{M}{\sin\alpha} + m\sin\alpha}$$

$$= -\frac{mg\cos\alpha\sin\alpha}{M + m\sin^2\alpha}$$

[compatible with UPII solution ☺]

Part B: (3) $\Rightarrow$

$$m a_x \overset{(1)}{=} -MA$$

$$a_x = +\frac{M}{m}\frac{mg\cos\alpha}{\frac{M}{\sin\alpha} + m\sin\alpha}$$

$$= +\frac{Mg\cos\alpha}{\frac{M}{\sin\alpha} + m\sin\alpha}$$

$$> 0$$

so the block slides down the wedge to the right!

Part C:

$$(4) \rightarrow \quad a_y = \frac{N \cos\alpha}{m} - g$$

$$\overset{(1)}{=} \; - \frac{MA \cos\alpha}{\sin\alpha \, m} - g$$

$$= + \frac{M}{m} \frac{\cos\alpha}{\sin\alpha} \frac{mg \cos\alpha}{\frac{M}{\sin\alpha} + m \sin\alpha} - g$$

$$= \underbrace{+ M \frac{g \cos^2\alpha}{M + m \sin^2\alpha}}_{a_y^0 \text{ (for later)}} - g \; > -g$$

so the block slides down the wedge at an acceleration less than $|g|$.

Part D: (1) changes to

$$MA' = -N \sin\alpha + F \qquad (1')$$

We want $a_y = 0$, so from Part C, we can write

$$a_y' \doteq 0 = - \frac{MA'}{m} \frac{\cos\alpha}{\sin\alpha} - g$$

$$= \frac{M}{m} \frac{\cos\alpha}{\sin\alpha} (N \sin\alpha - F) - g \quad < a_y$$

Oops, let's do this differently.

Similar to before, we write as "stationary condition"

$$N - mg\cos\alpha - mA'\sin\alpha = 0$$

$$-\frac{MA' - F}{\sin\alpha} - mg\cos\alpha - mA'\sin\alpha = 0$$

$$A' = \frac{-mg\cos\alpha + \frac{F}{\sin\alpha}}{\frac{M}{\sin\alpha} + m\sin\alpha}$$

Part B':

$$a_x' = + \frac{\left(Mg\cos\alpha - \frac{F}{\sin\alpha}\right)}{\frac{M}{\sin\alpha} + m\sin\alpha}$$

$$< a_x$$

Part C':
$$a_y' = -\frac{MA'\cos\alpha}{\sin\alpha\, m} - g$$

$$= a_y^0 - \frac{M\cos\alpha}{m\cancel{\sin\alpha}} \frac{\frac{F}{\sin\alpha}}{\frac{M}{\sin\alpha} + m\sin\alpha} - g$$

$$= a_y^0 - \frac{M}{m} \frac{F\cos\alpha}{\frac{M}{\sin\alpha} + m\sin\alpha} - g < a_y$$

$$\Leftrightarrow \quad F = \frac{m}{M} \frac{\frac{M}{\sin\alpha} + m\sin\alpha}{\cos\alpha} \cdot (a_y - g)$$

and I got really fed up with the question at that stage!

17/11/05

– Rdo